

Rounding Sum of Squares Relaxations

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I will survey recent developments on understanding the "Sum of Squares" (SOS) algorithm and its applications in several areas including graph algorithms, machine learning, and quantum information theory. This algorithm, which was proposed independently by researchers from different communities including Parrilo (2000) and Lasserre (2001), is a natural approach for nonlinear optimization. It is also closely connected to classical mathematical questions such Hilbert's 17th problem, that asks when can we certify that a polynomial is non-negative by writing it as a sum of squares. However, the running time of the SOS algorithm is exponential in the worst case, and there are fairly few cases where it is proven to run faster than that.

We will see a general approach for showing fast convergence for this algorithm, based on the algorithm's relation to the Positivstellensatz proof system of Grigoriev and Vorobjov (1999) and its view as optimizing over "fake marginals" of probability distributions. We demonstrate the usefulness of this approach by giving improved algorithms for several problems arising in machine learning and optimization, in particular finding sparse vectors in subspaces and learning sparse dictionaries. We'll also discuss the viability of this approach as a way toward refuting Khot's Unique Games Conjecture.

The talk will be mostly based on joint work with Jonathan Kelner and David Steurer.